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Clifford Allison Rose, Jr.  
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TIME MULTIPLEXING OF COMPENSATION

IN CONTROL SYSTEMS

by

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B. S., U. S. Naval Academy  
1959

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Chairman, Departmental Committee on Graduate Students



TIME MULTIPLEXING OF COMPENSATION  
IN CONTROL SYSTEMS

by ,

CLIFFORD ALLISON ROSE, JR.

Submitted to the Department of Aeronautics and Astronautics and to the Department of Electrical Engineering on August 21, 1967 in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics and the degree of Electrical Engineer.

ABSTRACT

The time sharing, or multiplexing, of amplification has been used for many years. However, there is a conspicuous absence in the literature of information regarding the multiplexing of elements used for dynamics, or to impart phase to a system. The purpose of this thesis was to investigate the feasibility of using a single compensation device with both gain and dynamics to stabilize several identical control loops. It is assumed that the form of the compensation is to be analog rather than digital.

The major effort of the analysis was directed from the standpoint of making the compensation available to each loop for a finite period of time, i.e. a sampled data system with finite pulse duration. Thus for a system with  $n$  loops, the compensation would be available for a particular loop for  $T$  seconds, then removed for  $(n-1)T$  seconds. A particular control system was studied, and analog computer simulation was conducted to determine appropriate compensation and sampling period.

Results of the simulation showed that it is possible for one compensator to stabilize several control loops. The modified compensation is similar in form to that required in the continuous loop, except that the time constants have been reduced by an amount proportional to the number of loops in the system. This reduction in time constants translates to a reduction in size of the associated capacitors, an important factor in this era of integrated circuitry. Unfortunately, some memory circuitry must be used in conjunction with the compensation. To perform its task properly, the compensator must

enter a loop with initial conditions reasonably close to the information it held when it was switched out of that loop. This requirement is a disadvantage, since information from one loop must be erased before the compensation enters the next loop if the individual systems are to be decoupled.

The thesis concludes with a discussion of the feasibility of implementation from the hardware point of view.

The final results of this investigation indicate that it is not feasible to multiplex both the compensation amplifier and the RC filter. It does seem that multiplexing the compensation amplifier may very well reduce the space required for a multiloop control system of identical controllers. The objective of improving reliability through reduction of components was not achieved to satisfaction. Although fewer amplifiers are needed, the amount of switching and circuitry exceeds that which had been desired.

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Title: Associate Professor of Aeronautics and Astronautics

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## Section I

### Introduction

The time sharing, or multiplexing, of amplification has been used for many years. For example, in a multichannel drift-stabilization system, one stabilization amplifier may be time shared by several dc amplifiers using a commutator which samples the error point voltage of each amplifier in turn. Any slowly varying voltage at the error point is applied to the stabilization amplifier as a series of pulses occurring at the pulse repetition rate of the commutator. These pulses are amplified and invested in the stabilization amplifier, and then applied to the dc amplifier through a holding circuit and a smoothing filter.

There is a conspicuous absence in the literature of information regarding the multiplexing of elements used for dynamics, or to impart phase to a system. The purpose of this thesis was to investigate the feasibility of using a single compensation device with both gain and dynamics to stabilize several identical control loops. It is assumed that the form of the compensation is to be analog rather than digital. Advantages which stand to be gained are:

- 1) A reduction in the number of components in a system often improves its reliability.
- 2) Elimination of redundant elements would yield additional space inside a module, a factor extremely important in aerospace systems. Techniques in integrated circuitry have reduced the size of

transistors and resistors, but not capacitors. Therefore, reduction in the number of capacitors would greatly alleviate the packaging problem.

The control system studied was a typical temperature regulator, with controller, sensor, compensation and heater. The block diagram in Figure 1 describes the system.

A step disturbance of one watt is applied. The sensor, a thermistor bridge, samples the temperature from the controller and compares it with the reference, or quiescent, temperature  $T_Q$ . The difference is transmitted to the compensator, where appropriate gain and phase are applied. The resulting signal is sent to the heater, which supplies the nominal quiescent heating  $W_Q$ , plus an increment to correct any error. (The nominal heater voltage is one-half the maximum value, thereby supplying power continuously to account for ambient losses, and allowing maximum dynamic range to counteract disturbances.)

Simplifying this block diagram and inserting transfer functions, Figure 2 is obtained.

For purposes of the thesis, it is assumed that there are several control loops with identical controllers. The objective will be to determine if the arrangement in Figure 3 will work satisfactorily.

The compensator is to be made available to all loops in sequence. When the compensator is switched out of the loop, its last value is kept on the heater by the hold. The design problem is to determine the appropriate gain, compensation dynamics and sampling

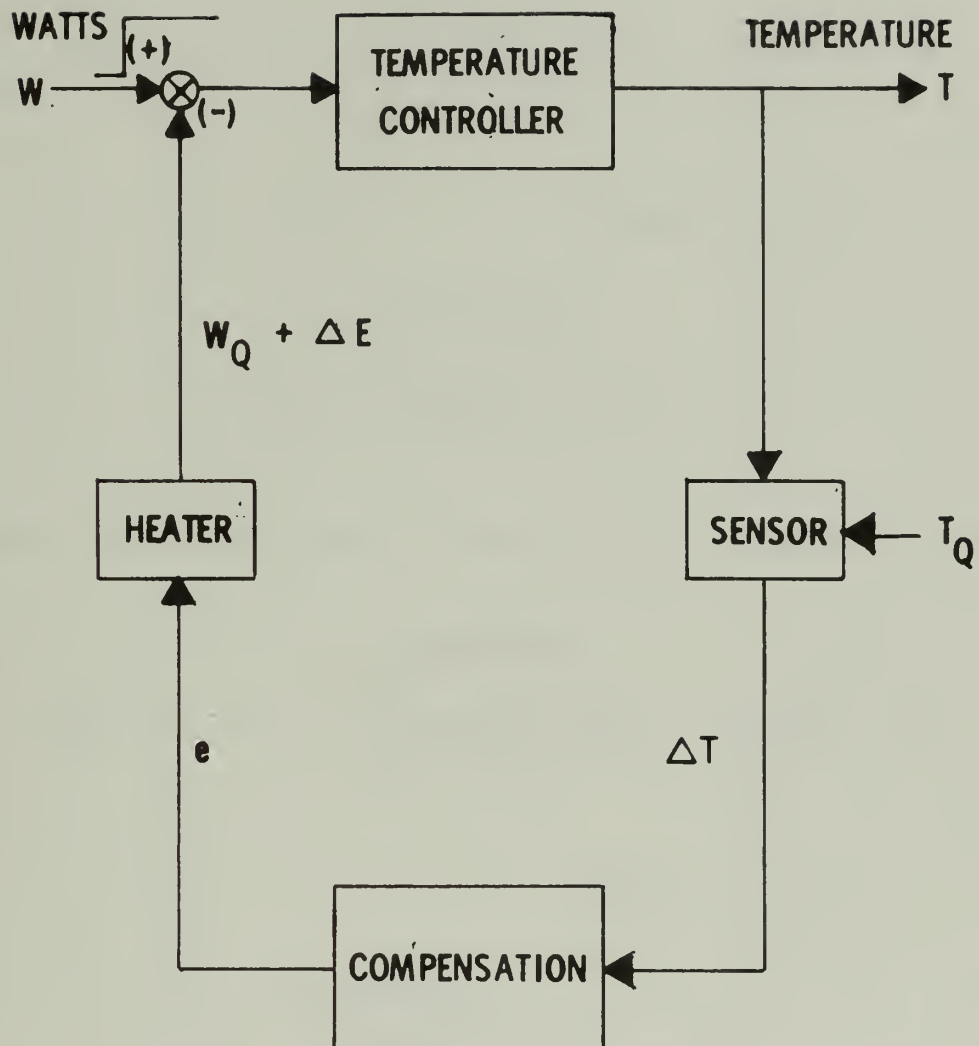


Fig. 1. Block diagram of temperature control system.

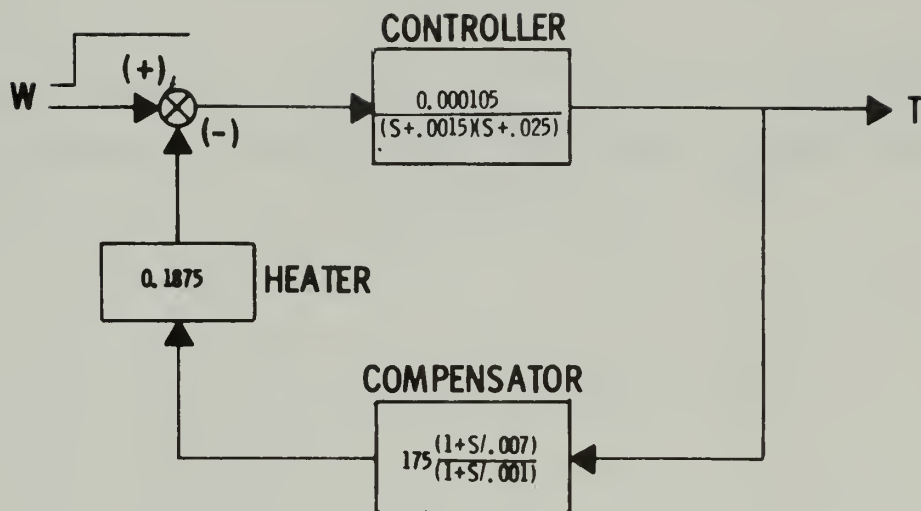


Fig. 2. Simplified block diagram for temperature control system.

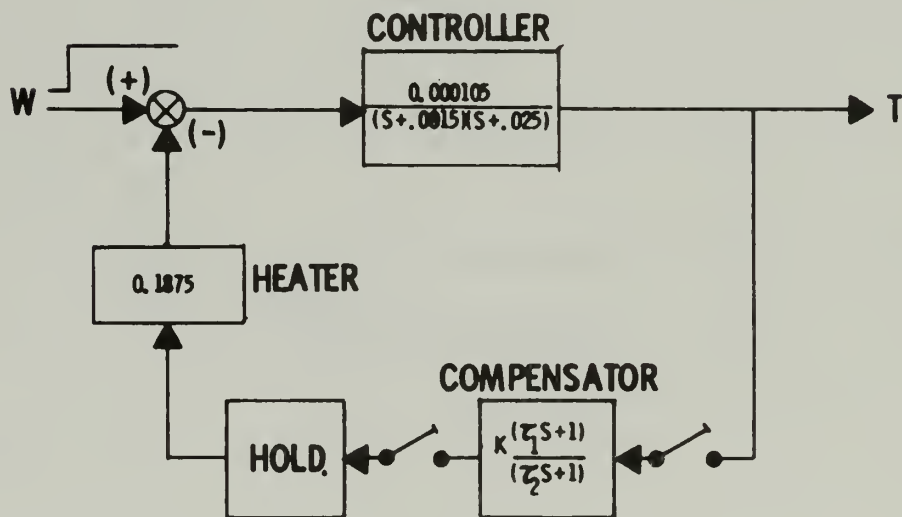


Fig. 3. Block diagram of system permitting time sharing of compensation.

period which will result in an acceptable transient response of the controller. If an analysis indicates that it is possible for such an arrangement to perform satisfactorily, then a study will be made of the difficulties in implementation.





## Section II

### Methods of Analysis

Modern control theory would suggest that the problem be first analyzed using sampled data theory. Sampled data texts contain numerous examples of control loops with a device similar to the unit labeled compensator in Figure 3, referring to it as a digital controller. Texts also treat adequately the manner in which digital compensation may be realized using conventional resistors and capacitors. Unfortunately, no simple scheme to realize the controller in a multiplexing arrangement using resistors, capacitors, and data holds could be devised. The following explanation reveals why.

The usual sampled data analysis assumes that a signal is sampled during an infinitesimally small period of time, and that this value is held during the remainder of the sampling period. Some holding device must then be employed after each switch. Figure 4 shows how a zero order hold reconstructs a signal  $T(t)$ . Naturally, if the compensation device is to be time shared in several loops, it must be separated from the remainder of the loop by a pair of switches. This requirement means that a hold must be installed in each loop after the compensation, and a hold for each loop must be present at the input of the compensator. The hold in the feedback of each loop serves to keep the heater on when the compensation is switched out. The holds at the input enable the compensator to reconstruct the output of each controller before adding the necessary gain and phase.

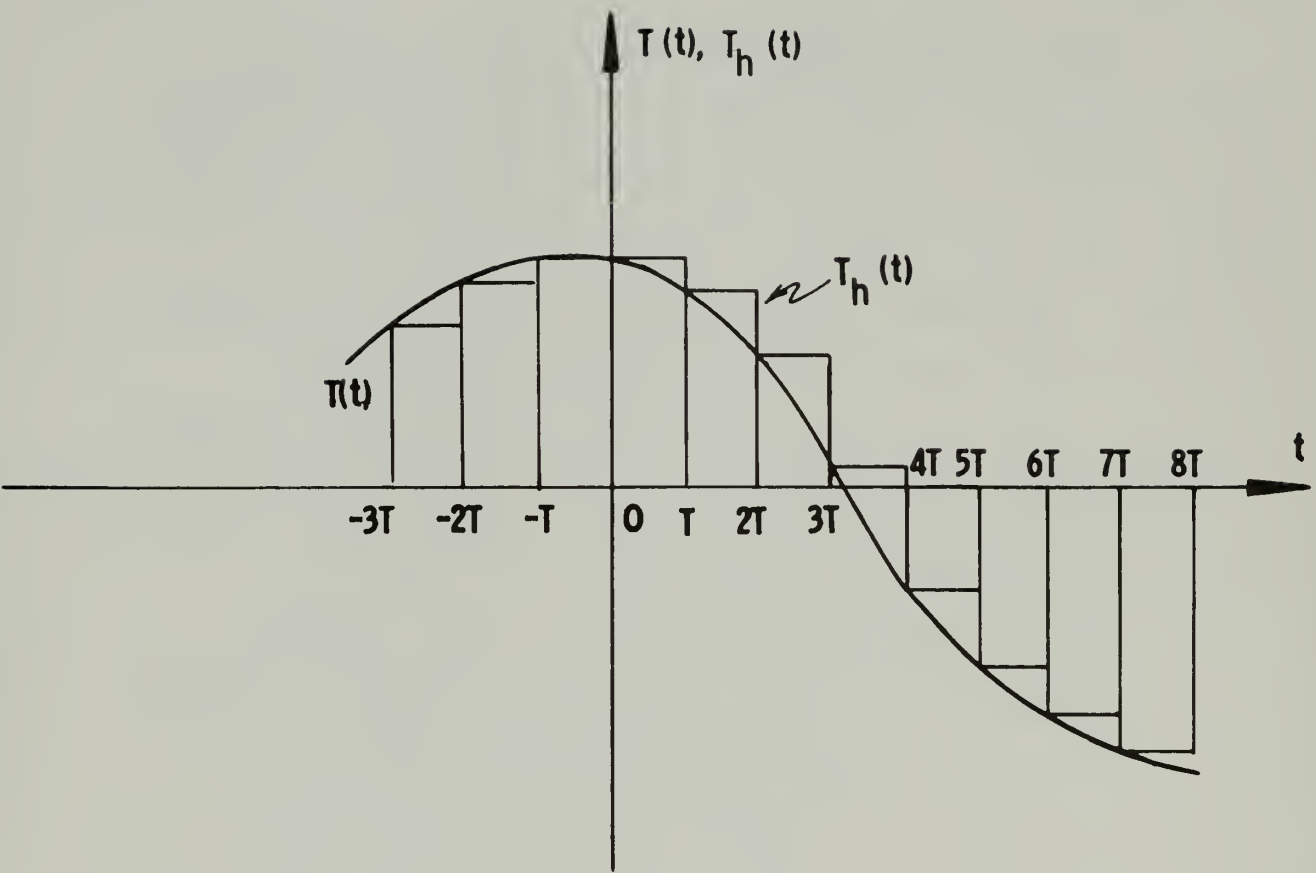


Fig. 4. Reconstruction of  $T(t)$  by  $T_h(t)$  using zero order hold.

One cannot rely on the RC filter circuit used for dynamics in the compensation to perform this task. In order that the loops be decoupled, capacitors with voltages from one loop must be discharged before being inserted in the next loop. But from Figure 4 it is clear that when the compensator returns to loop #1, it must return to the last value it had before being switched out of that loop. At the  $n^{\text{th}}$  sample in a loop the compensator must "remember" its value during the  $n^{\text{th}}-1$  sample. One is faced with the alternative of using a separate memory channel in the compensator for each loop, or devising a method to transfer this information from the hold in the feedback of each loop to the compensator. The latter arrangement would require switching and an amplifier to insure that the voltage on the smaller hold capacitor is transferred completely to the larger capacitors in the filter circuit. One should now appreciate why the function of a digital controller shared between several loops is normally performed by a digital computer or some logic circuitry, with a separate channel for each control loop.

The requirement for memory, with its associated hardware, diminishes the attractiveness of utilizing one compensation device for several loops. However, sampled data theory assumes that the time the switch is closed is very small compared to the dominant linear system time constants. In an effort to avoid the complexity of memory circuitry, the major effort of the analysis was directed from the point of view of using a finite pulse width.

A desirable form of the compensation would be just an amplifier for gain and an RC filter for dynamics, without any hold circuits. During the interval of time that the compensation is in each loop, the amplifier would apply gain and the dynamics would impart phase. This method implies that for a system with  $n$  loops, the compensator would be available for a particular loop for a pulse duration of  $T$  seconds, then removed for  $(n-1)T$  seconds. The reasoning behind this approach was if the compensator were in a loop for a suitable period of time and if the time constants of the compensation were modified from the values used in the unshared system, perhaps enough regulation could result from the time that the compensator spent in each loop. In other words, it was hoped that a finite pulse duration would enable the capacitors of the compensation to be discharged during switching, and avoid the necessity of installing separate memory channels for each loop. Of course, a hold in the feedback of each loop would still be required.

Should this method prove satisfactory, hardware needed for memory could be eliminated and one amplifier with associated RC filter could be shared among the systems. An additional feature would be that intuitively one might expect that the modification of time constants would consist of speeding up the dynamics, to account for the fact that the compensator is available for a shorter period. A fortunate result of decreasing the time constants is that smaller capacitors would be used, also contributing to reduction in size of the regulator module.

At this point it became necessary to select a suitable method of analysis that would permit determination of the required compensation and selection of a suitable sampling period. At least two methods are available. The first is an extension of sampled data theory, where the  $z$  transforms for infinitesimal switch closures are replaced by the  $p$  transform for finite pulse width. Sampled Data Control Systems by E. I. Jury devotes a chapter to this subject. As one might expect, the design procedure is considerably more complicated than that involving  $z$  transforms. In the latter case, for a given sampling frequency, the transfer functions in terms of the Laplacian operator are transformed to a rational polynomial in  $z$ . Except for unusually complicated systems, it is generally possible to select the compensation for the desired response without resorting to the cut and try process by using conventional design techniques in the  $z$  plane. However, when using  $p$  transforms it is not possible to get a corresponding rational polynomial in  $p$  due to the pulse width factor. The design procedure here would be to write a digital computer program to determine the time response of the system, with parameters of sampling period, pulse width, compensator gain, and compensator pole-zero configuration.

An alternative procedure would be to use the Laplace transform and determine the transient response on a piecewise linear basis. A pencil and paper solution to the problem would be extremely laborious, since the initial condition terms in the Laplace transform are not zero as they so often are. To illustrate, for the period when the compensator is in the loop the initial conditions are determined by solving the

differential equation for the system with the compensator out, then evaluating at the time it is switched in. These equations are hardly suitable as design equations. It was decided then to use analog computer simulation, electronically solving these sets of equations. The effect of varying compensation and sampling period may be conveniently displayed on an oscilloscope.

### Section III

#### Analog Computer Model

Standard references on analog computation proved to be of little help in formulating the desired model. Nearly all texts discuss the derivation of the model from the differential equation describing the system, which is not the most convenient for a piece-wise linear problem.

In this case it was much easier to obtain the computer model from the separate transfer functions in the block diagram of Figure 3. This procedure proved to be quite easy; most of the trouble experienced was in achieving proper magnitude scaling.

Correct magnitude scaling is necessary if the amplifiers and integrators are to operate in a linear fashion. Texts treat the problem in two ways. One method relies on a knowledge of the physics of the problem to accurately estimate the maximum values of the physical quantities involved. For an unfamiliar problem this approach can be deceptively difficult. Usually computer simulation is employed because existing knowledge of the system is insufficient. The second alternative is to use various thumb rules, which seem to work well in textbooks and poorly everywhere else.

The following method was suggested by Mark Connelly of the Electronic Systems Lab. It uses the well known approximation that for control loops with high open loop gain, the steady state value of the



output is very nearly the reciprocal of the feedback gain. Applying this concept to the individual integrators and allowing for overshoot, scaling was accomplished in a straightforward manner. Derivation of the analog computer model illustrated in Figure 5 is described in the following paragraphs.

The lower break frequency of the controller is 0.0015. This factor is realized by using an attenuator of 0.15 in the feedback of an integrator with gain 0.01. If this integrator is preceded by another potentiometer with setting of 0.15, then the steady state response of the integrator to a unit step is unity.

The same procedure is followed for the second integrator; with one modification. The break frequency specifies an attenuator of 0.25 in the feedback, and this scaling procedure suggests a potentiometer in front of the same value. However, the gain of the controller imposes the constraint that the forward path gain of both integrators must be 0.000105. Input gains of ten in the through path of each integrator were found to be necessary for the loop gain of the model to agree with that of the system. Integrator gains of 0.01 and 0.1 are required for the proper time constants. Therefore, it appears that the potentiometer in front of the second integrator is constrained to be 0.007 instead of the desired 0.25. These contradictory demands may be resolved by employing a setting of 0.25 on the potentiometer, and realizing that the output of the controller model is 35.7 times greater than the corresponding output of the real controller.



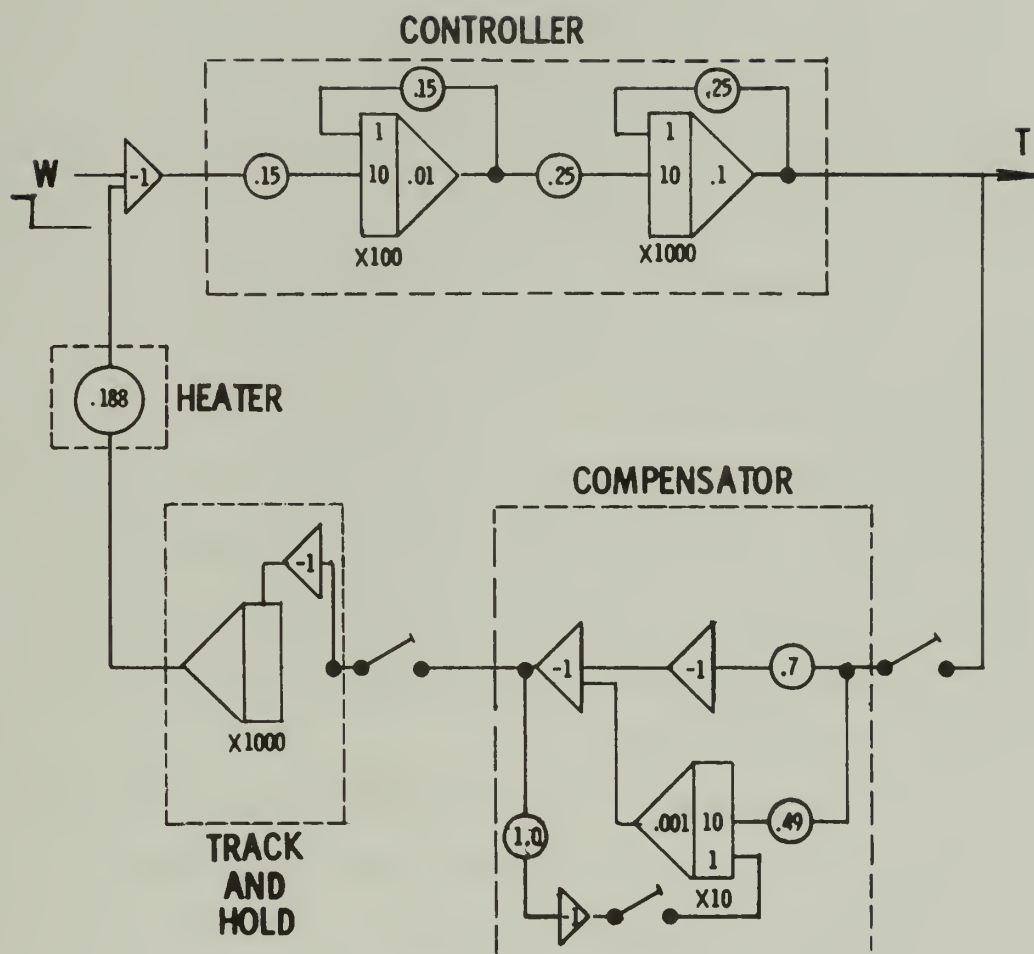


Fig. 5. Analog computer model.

The model for the compensator was obtained readily from the transfer function. Numerical quantities in Figure 5 correspond to the compensation in the existing system. As a result of the above scaling for the controller, the gain of the actual compensator must be divided by 35.7 for modeling.

The track and hold is a device which has no effect when the compensator is in the loop, but holds its last value when the compensator is switched out. It is implemented by directing the output of the compensator into the initial condition jack of an integrator rather than the conventional input. The heater is modeled by a potentiometer. Various inverters are required for correct algebraic sign. This sign reversal for an amplifier or integrator requires that the input be a negative step.

Time scaling was necessary in order that the solution time of the problem be within the capabilities of the oscilloscope. It can be shown rigorously by substituting a change of variables into the governing differential equation that changing the time scale is equivalent to multiplying the integrator gains by the appropriate constant. For this problem to permit oscilloscope display, all integrators must be multiplied by 10,000. Resulting gains are as indicated in Figure 5 by the factor beneath the integrator. The integrator gain for the track and hold may be selected arbitrarily, so the maximum available gain of 1000 was chosen to minimize the amount of decay during the hold phase.

The transient response for the existing system was known. Wiring and scaling procedures were checked by leaving the switches closed and modeling the present compensation. When the transient response of the model with the present compensation agreed with the known response, full scale simulation was begun.



## Section IV

### Computer Simulation

The reader will recall from the discussion in Section II that to be competitive with continuous systems, the multiplexed compensation must operate as a sampled data system with a finite pulse duration. It was felt that during the pulse, or length of time that the compensator was in each loop, the dynamics of the compensation could impart enough phase for satisfactory regulation without the use of separate hold circuits. The first step in the simulation was the selection of a suitable pulse width. Experience with conventional sampled data systems using instantaneous sampling had shown that the sampling frequency must be high enough to avoid introducing an excessive amount of lag. For this problem the sampling period is an integer multiple of the pulse duration, where the integer represents the number of controllers that could be served by a single compensator. This constraint would suggest a fairly narrow pulse width. Since the dominant time constant of the closed loop response with unshared compensations is 110 seconds, it was decided to use a pulse duration of ten seconds.

This particular analog computer, the GPS 200T, did not have sufficient flexibility in switching and synchronizing pulses to allow more than one loop to be modeled. Therefore, for this simulation, the compensation must be discharged and reinserted in the same loop, however, the equivalence of this arrangement and a multiloop system with identical

plants is obvious. Since behavior of the system was unknown, it was decided to restore the present compensation each cycle for ten seconds; and gradually increase the length of time the compensation was out of the loop. Hopefully, the length of time the compensator could be kept out would be an integer multiple of ten seconds.

The disappointing results of the first run are illustrated in Figure 6. (All figures in this section were reproduced from oscilloscope photographs. The oscilloscope was calibrated to remove the scaling factor of 35.7 mentioned in the previous section.) A comparison of the two waveforms shows that the servo system with discharged compensation does not act to reduce effectively the error between desired and actual temperatures. No regulation occurs unless the compensator is supplied the difference between the present output of the controller and its output the last time the compensation was in the loop. The effect is as though gain alone was used. Therefore, even if a finite pulse width is used, the compensation must reconstruct the output of the controller, as in Figure 4. Variation of pulse width did not correct this deficiency, as is to be expected in the light of the above reasoning. Therefore, the only recourse was to accept the requirement for memory in the compensator.

Figure 7 indicates the transient response of the controller if the present compensation is reinserted in the loop with the capacitor voltages maintained during the out period. To simulate this, it was found necessary to add the extra switch in the feedback path of the integrator (see Figure 5) in order that it fully hold its last value.

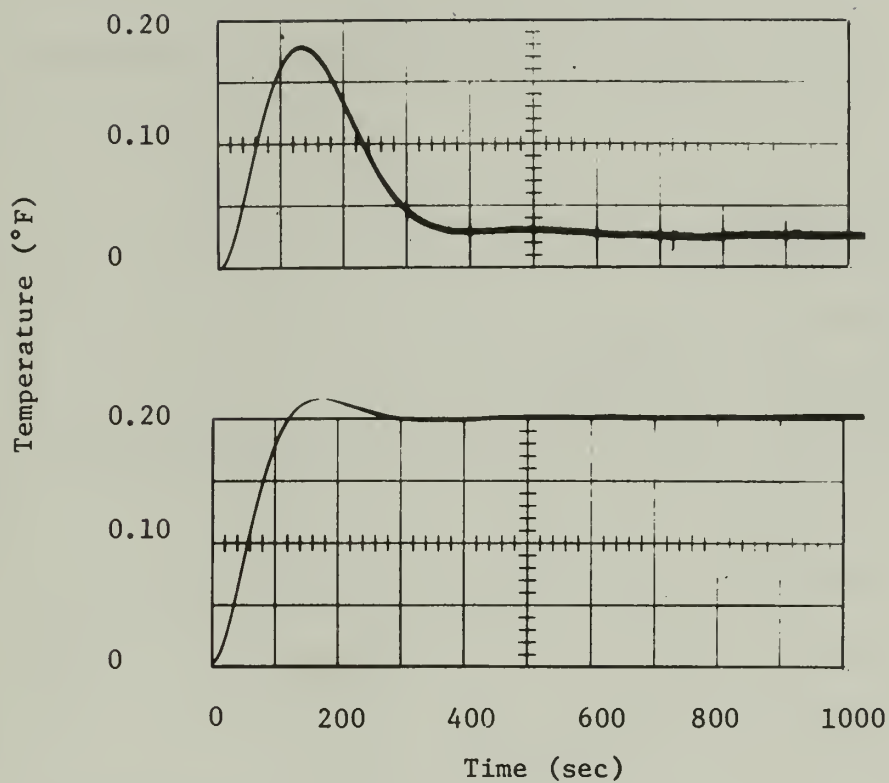


Fig. 6. Effect of memory on compensation. Top graph shows transient response of controller in continuous system. Lower graph illustrates response when compensation is switched out, discharged, and reinserted with zero initial conditions.

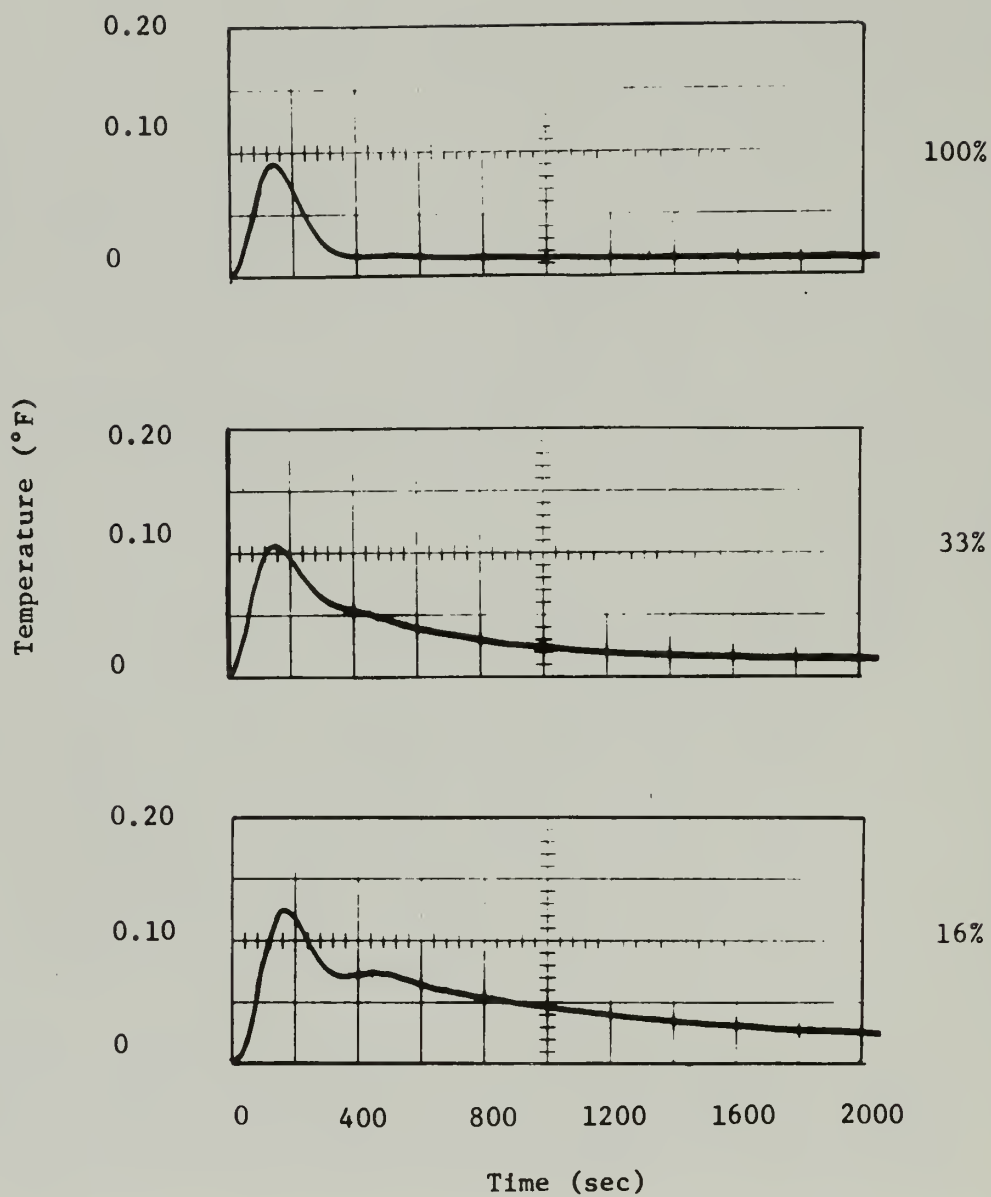


Fig. 7. Transient response of controller with present compensation made available for the indicated percentage of time. Compensator restored to its previous value. Pulse width ten seconds.



The middle graph illustrates the response if three controllers use the same compensation, with existing gain and pole/zero configuration. The lower graph shows the response of a disturbed controller if five other controllers share the same compensator. Although the response is not as sharp as it is in the continuous case, with memory it is at least workable.

The next step is obviously to seek improvement by using different compensation and/or pulse duration. Of course, there is no one correct solution, nor had a criteria for optimization been specified for the existing system. Therefore, a wealth of data on possible solutions could have been assimilated. However, it was the objective of this thesis to examine the problem from the systems point of view. Within the time limitations imposed, it was considered more desirable to obtain a sufficient amount of data to be considered representative of the behavior of the system, then to examine the difficulties in implementation; rather than a study in depth of the characteristics of loops with multiplexed compensation.

In determining a more suitable design, the pole/zero configuration of the RC filter was considered the primary parameter. The reasons for this choice are as follows. Since high gain and a desirable system frequency or transient response are usually conflicting demands, the gain is normally that just required to meet the maximum steady state error specification. It was on this basis that the value of gain was selected in the original system. By keeping this same value of

gain, the multiplexed system still meets this error specification, yet clearly indicates the behavior of multiplexed dynamics. Secondly, based on work in sampled data systems, it was believed that a sufficiently short sampling period would have a secondary effect.

Examining Figure 7, it is clear as the out time of the compensator increases, the response of the controller becomes more sluggish. Reasoning physically, the time constants of the original compensation are too slow to perform properly if the compensation is made available for a reduced period of time. The natural solution is to decrease the time constants, with results as shown in Figures 8 through 13.

The first three graphs show the response of a disturbed controller with the compensation in the loop ten seconds out of each thirty. A memory arrangement is employed, such that the compensation enters each loop with the appropriate initial conditions, namely the final conditions for the last period it was in that loop. The multiplicative factor beside each curve indicates the increase in pole frequency. Recalling that the compensation in the continuous system is  $175 \frac{(1 + s/.007)}{1 + s/.001}$ , the form of the compensation used in each experiment may be easily determined. For example, the top graph in Figure 8 shows the step response of a disturbed controller when compensation of the form  $175 \frac{(1 + s/.010)}{1 + s/.002}$  is shared among three regulators. It was necessary to suppress somewhat the parameter of pole/zero ratio. To give a better basis for comparison with unshared compensators, pole/zero ratios in the vicinity of the existing compensation were used.

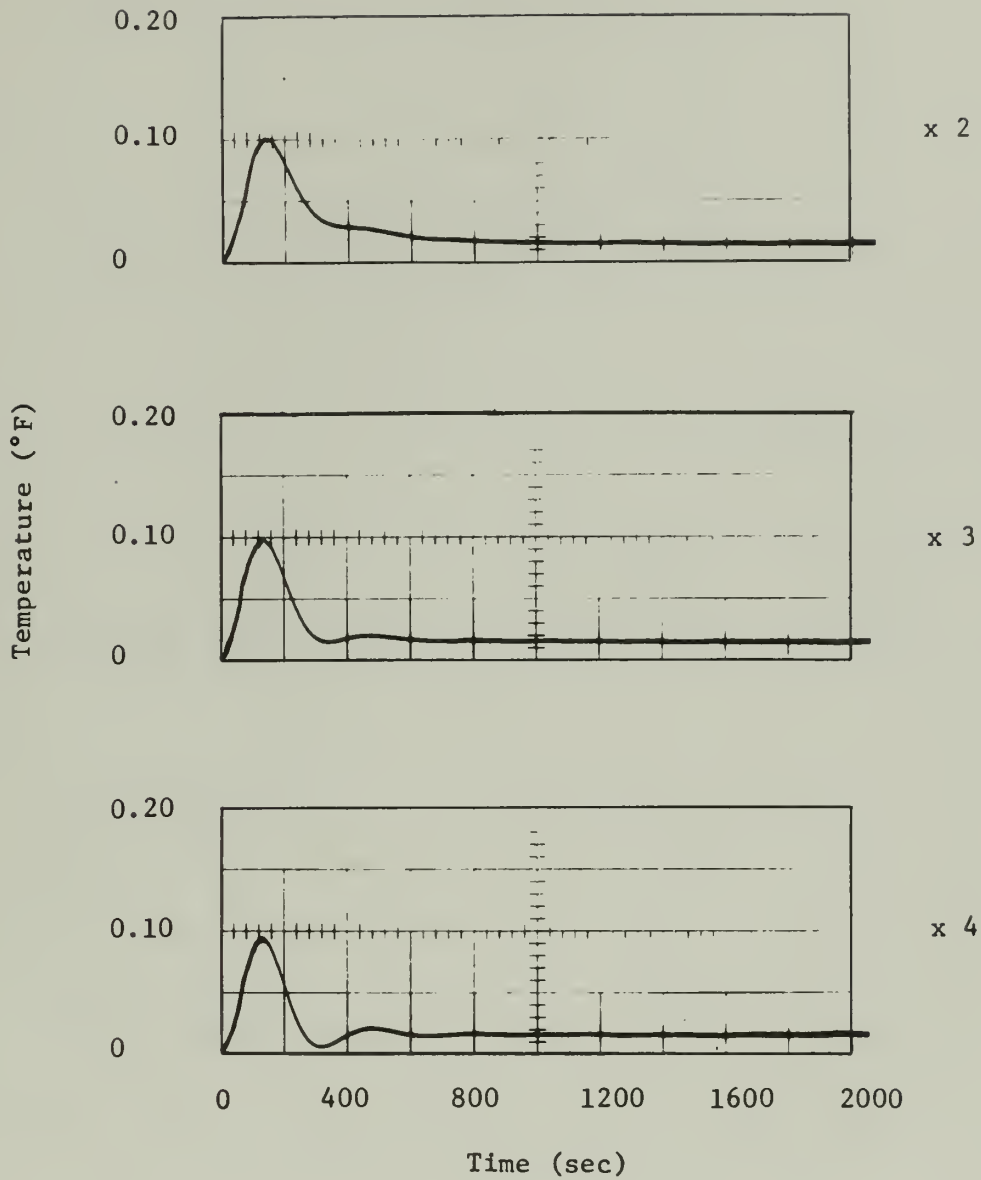


Fig. 8. Transient response of temperature controller, illustrating effect of increasing pole frequency while maintaining zero/pole ratio of 5. Compensation with memory used in three loops.

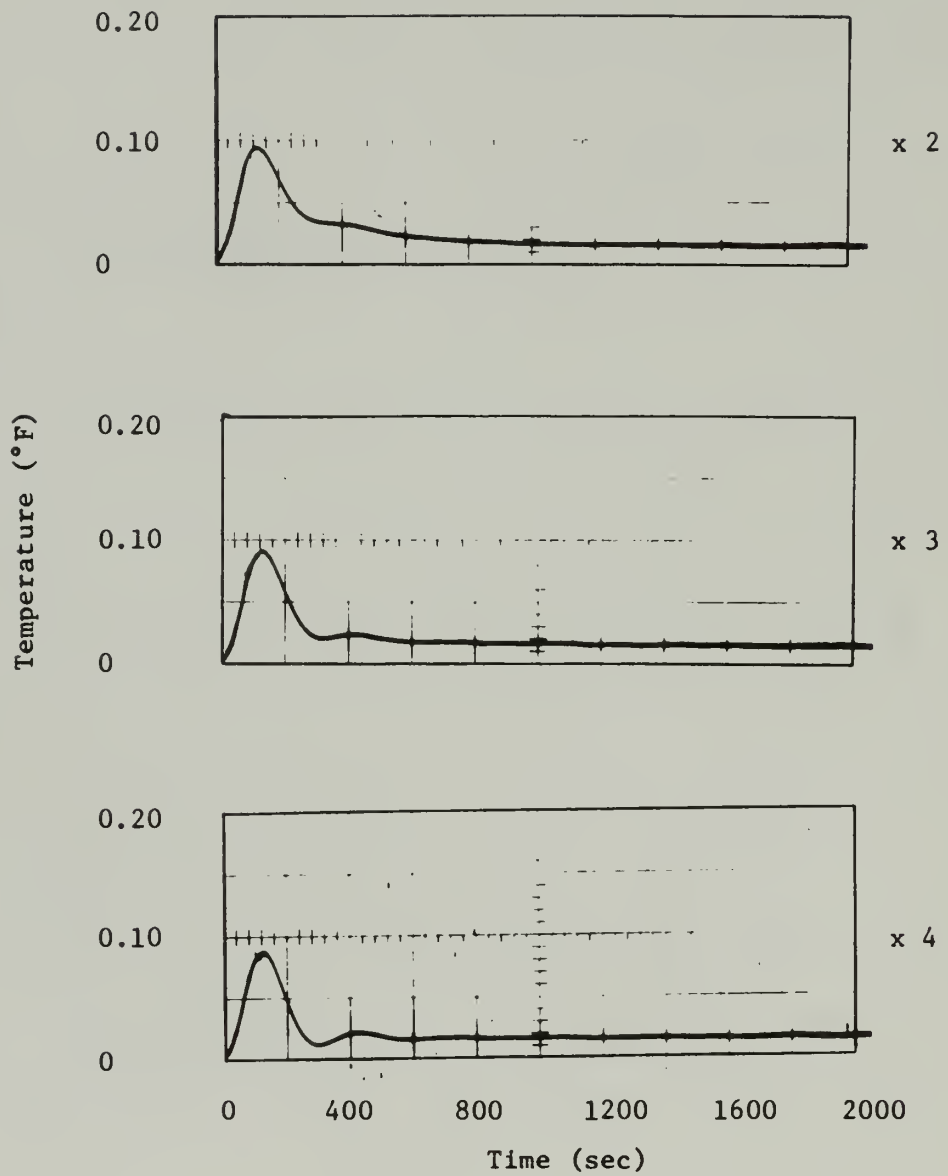


Fig. 9. Transient response of temperature controller, illustrating effect of increasing pole frequency while maintaining zero/pole ratio of 6. Compensation with memory used in three loops.

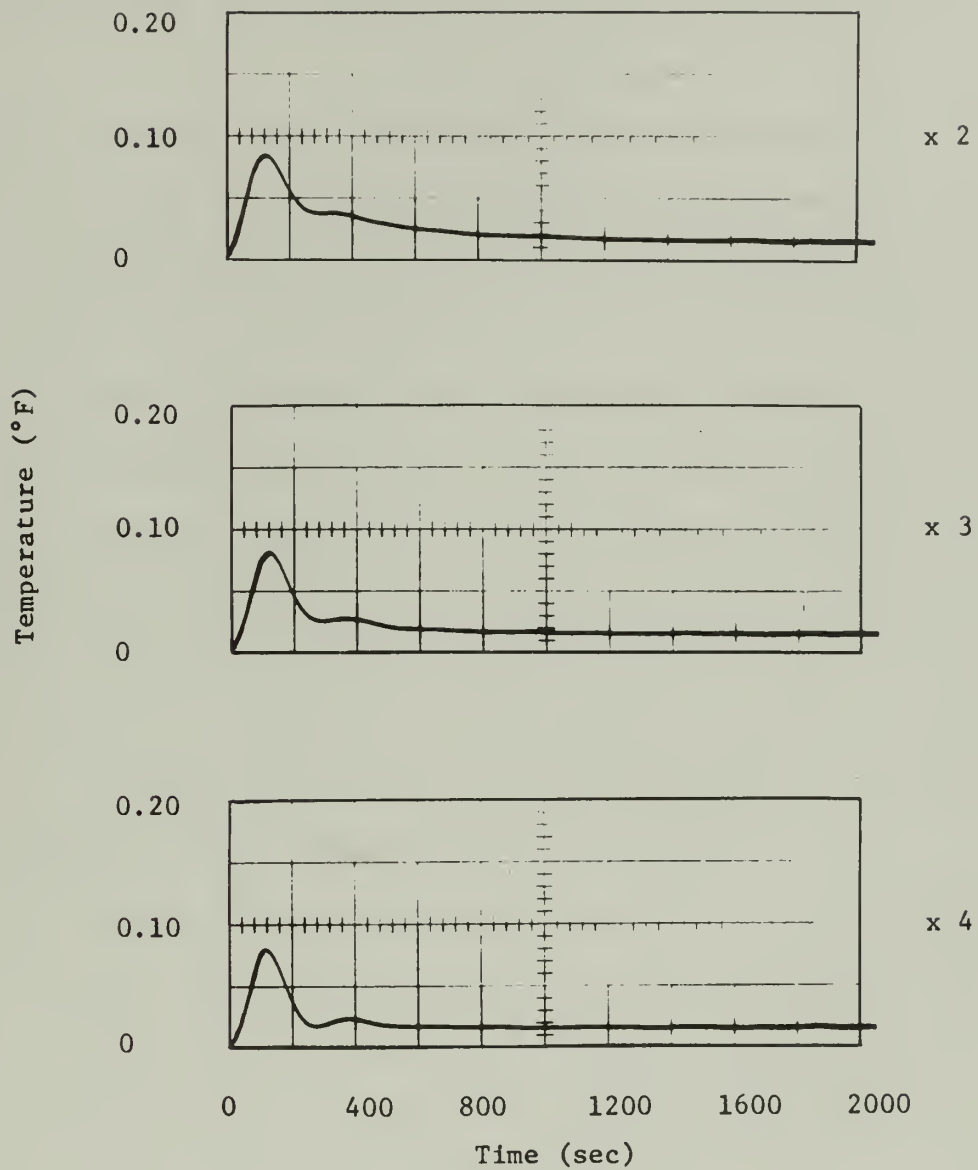


Fig. 10. Transient response of temperature controller, illustrating effect of increasing pole frequency while maintaining zero/pole ratio of 7. Compensation with memory used in three loops.

The improvement of several of these responses over that exhibited in the middle graph of Figure 7 is obvious.

Figures 11, 12 and 13 illustrate similar results if the compensation is simulated to be multiplexed in six loops. A comparison of the two series reveals that for the ten second pulse duration, as the compensator is multiplexed in more loops some deterioration in response results in spite of the changed compensation configuration. The amount of overshoot of the first peak is increased and ringing is more pronounced. However, as long as some memory scheme is employed, the transient response of a controller with compensation available for one-sixth of the time still compares favorably with the transient response of the same controller when compensation is available continuously. This point is demonstrated in Figure 14.

One will notice the somewhat cleaner response in the bottom graph of Figure 14 as the sampling period was shortened for a six system simulation. The smooth response using a pulse duration of five seconds could not be duplicated with a pulse of ten seconds, regardless of the pole/zero configuration. Therefore, as the number of plants sharing the same compensation increase, besides decreasing the time constants of the compensator, the designer should also consider reducing the sampling period. Of course, there may be occasions when the sampling period is constrained. Then the designer will have less freedom of choice about the form of the compensation. Figure 15 illustrates the not too surprising point that compensation which seems

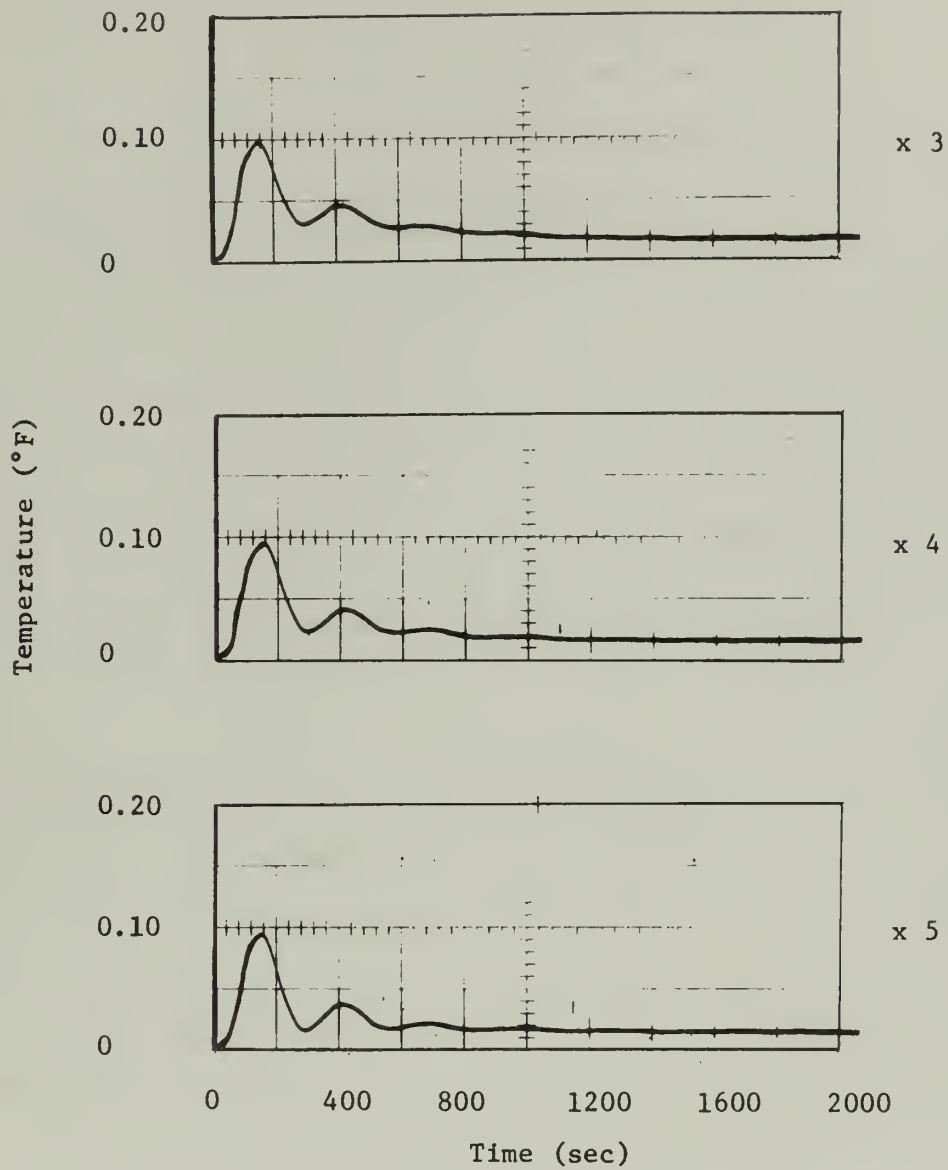


Fig. 11. Transient response of temperature controller, illustrating effect of increasing pole frequency while maintaining zero/pole ratio of 5. Compensation with memory used in six loops.

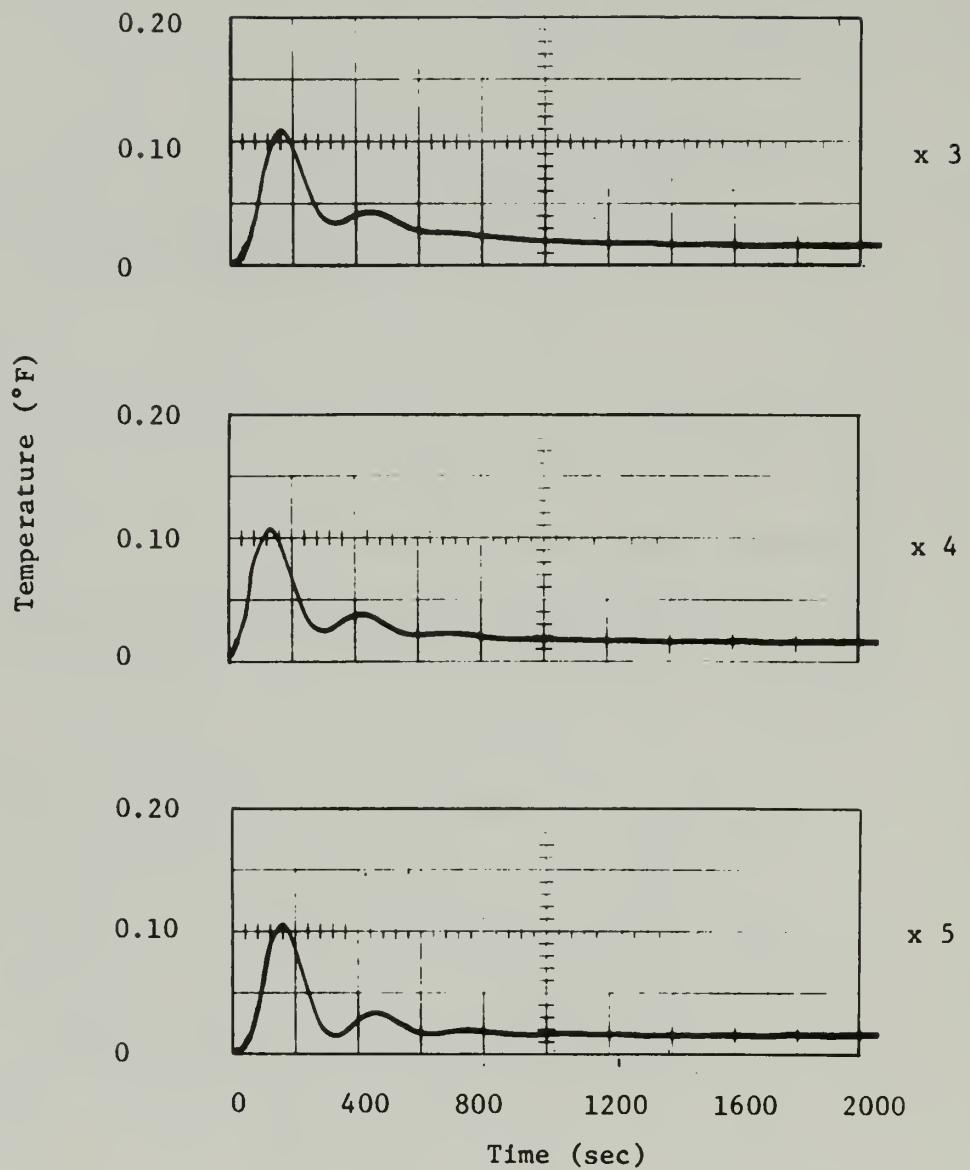


Fig. 12. Transient response of temperature controller, illustrating effect of increasing pole frequency while maintaining zero/pole ratio of 6. Compensation with memory used in six loops.



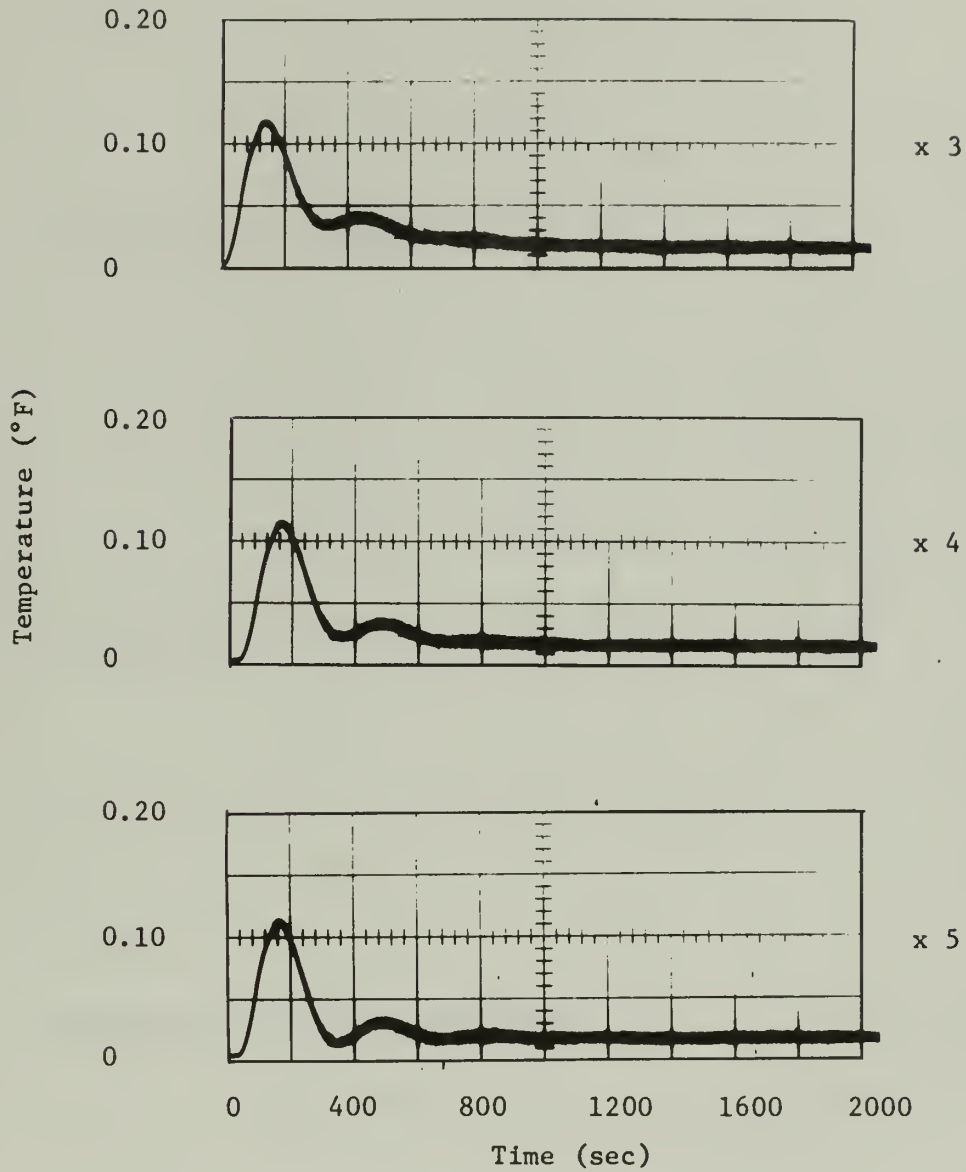


Fig. 13. Transient response of temperature controller, illustrating effect of increasing pole frequency while maintaining zero/pole ratio of 7. Compensation with memory used in six loops.

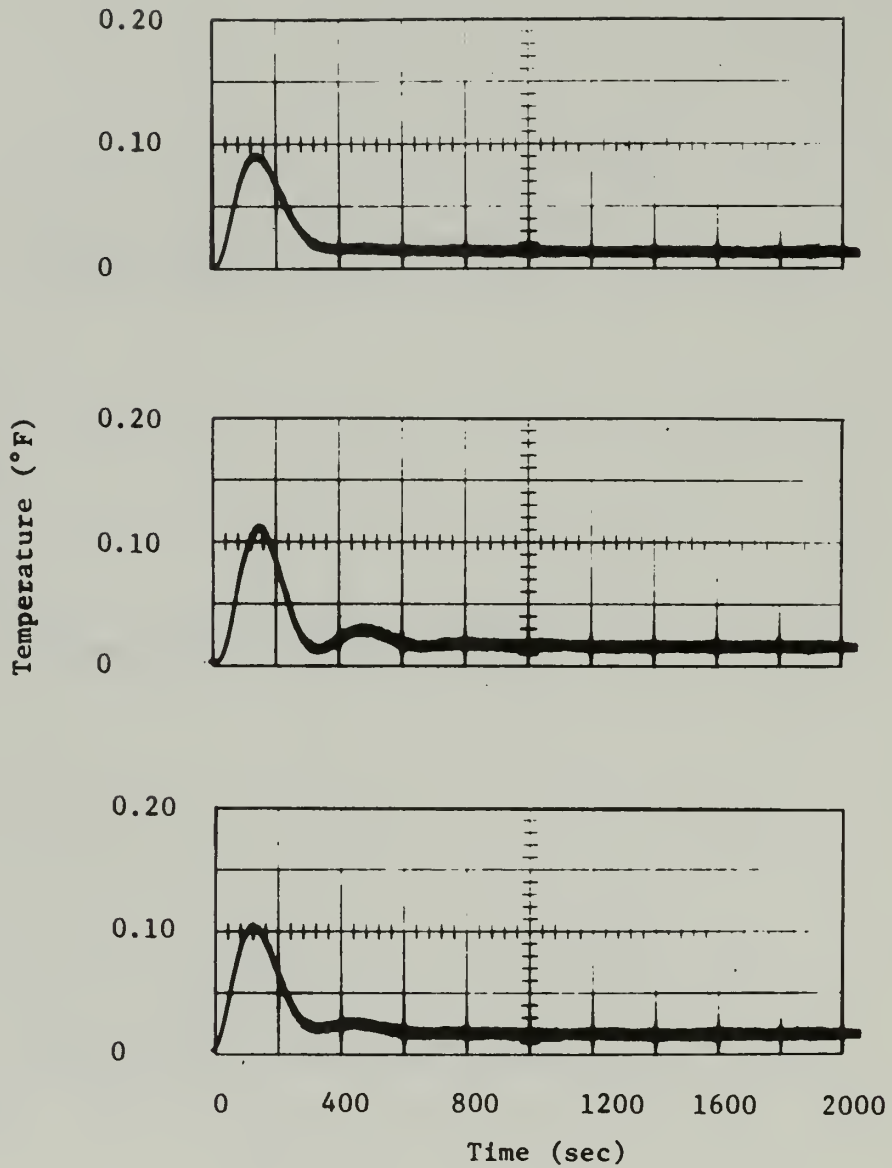


Fig. 14. Comparison of transient responses of controller. Top graph shows response of existing system. Middle graph illustrates response when modified compensation with memory is used in six loops for ten seconds each. Bottom graph shows response for pulse duration of 5 seconds.

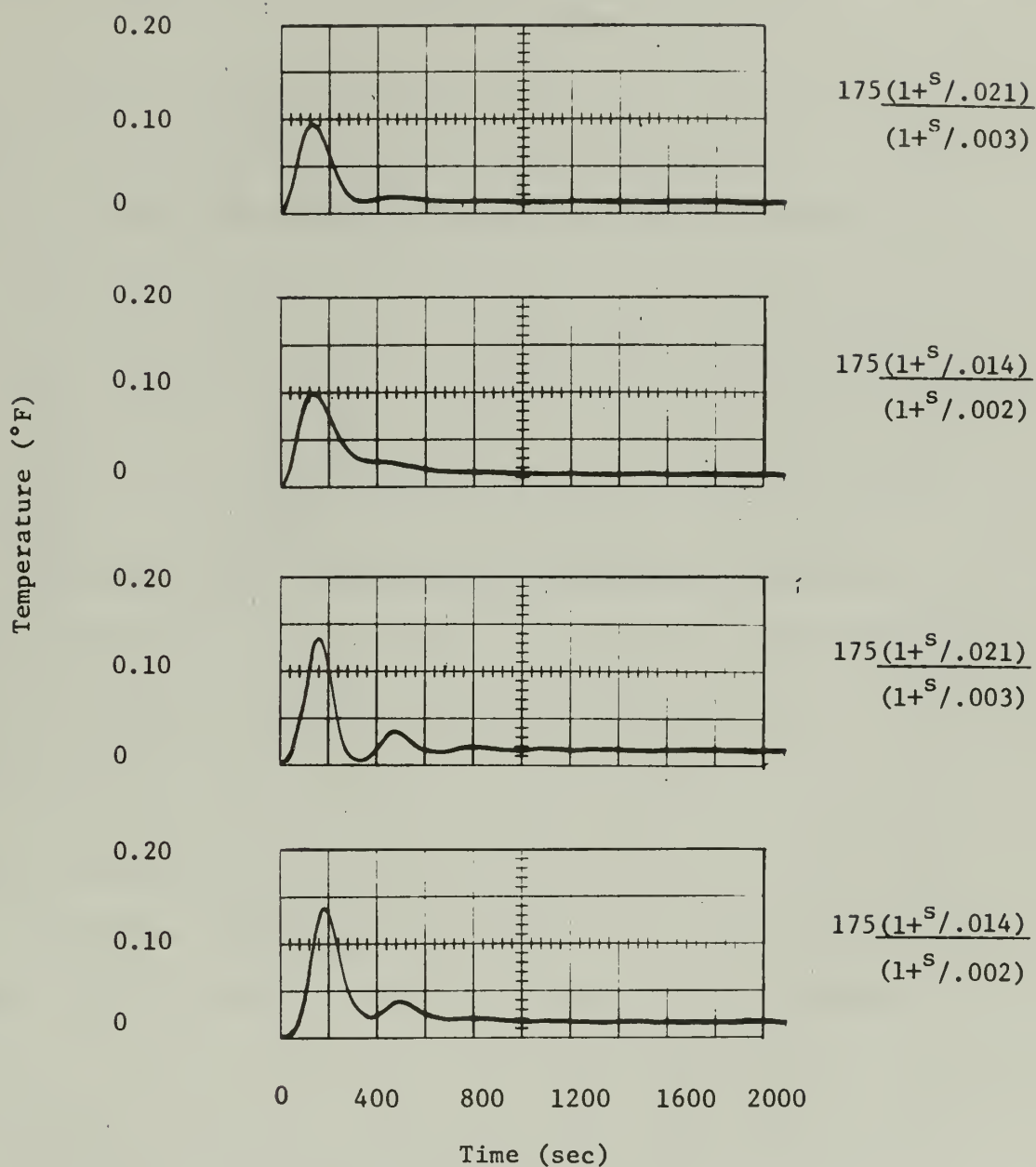


Fig. 15. Effect of pulse duration on choice of compensation. Pulse width for top two graphs 10 seconds; lower two graphs 20 seconds. Compensation as indicated is multiplexed in three control loops.

more desirable for one sampling period may very well be less desirable with a different pulse width.

Figures 16 and 17 show the close correlation between the effect of varying sampling period in this system and in systems using instantaneous or impulse sampling. Decreasing the sampling period in this problem by an order of magnitude did not grossly alter the performance of the system, just as in impulse sampling systems for sufficiently small periods. Similarly, if the period were doubled or tripled, performance using the same compensation became marginal.

The analog computer simulations indicate that it is possible to multiplex compensation in several control loops, provided that when the compensation enters a loop it has very close to the same information that it did when it was switched out of that circuit. In order that each system be decoupled from the others, information from one loop on the compensator must be removed before it is switched into the next loop. Therefore, some memory arrangement must be devised such that past voltages are restored to the capacitors of the compensation. This requirement for memory does mean that more components will be needed than had been originally intended. However, the end result may still be more compact than is presently the case with separate compensation for each loop, especially since the decrease in the time constants of the compensation results in a reduction in size of the associated capacitors.

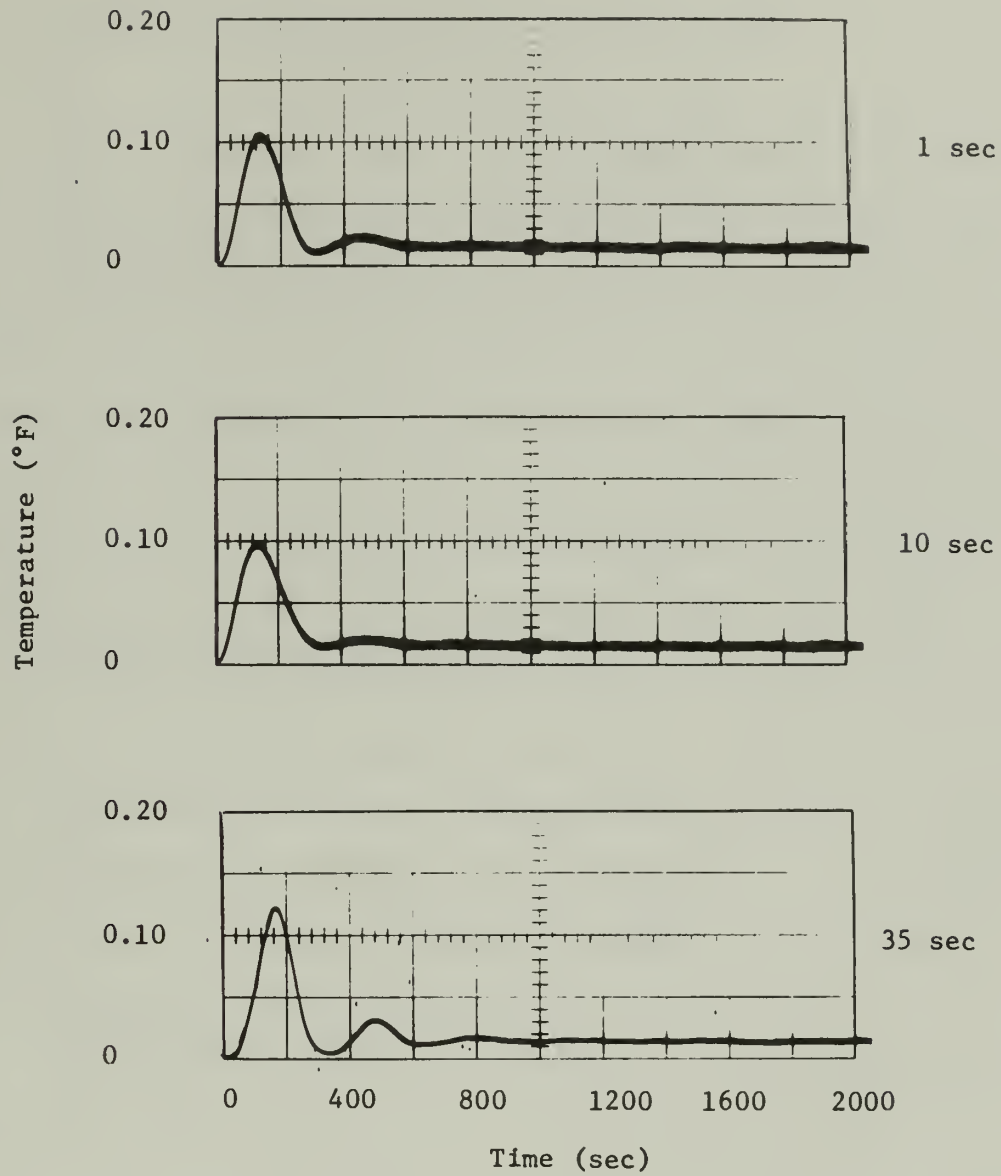


Fig. 16. Effect of pulse duration on transient response of controller. Compensation with memory is used in three loops.

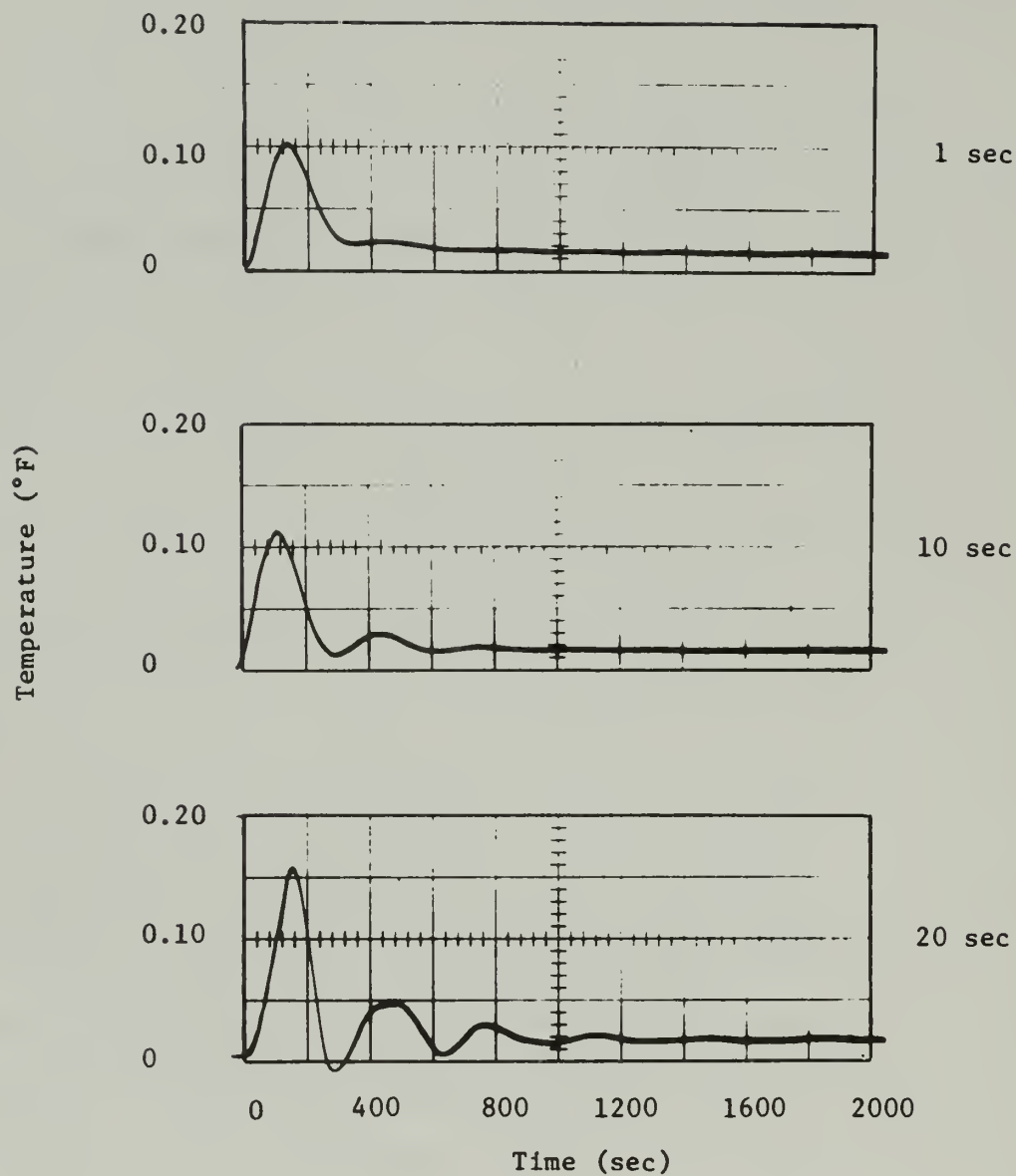


Fig. 17. Effect of pulse duration on transient response of controller. Compensation with memory is used in six loops.

## Section V

### Implementation

This study of multiplexed compensation would not be complete without an investigation of the hardware required for implementation. In fact, the entire research has been carried out with the implementation in mind. The concept of using a finite pulse width for the introduction of phase to the system originated from the desire to avoid the hold circuitry necessary in impulse sampling. Unfortunately, computer simulation showed that memory is still required even with a finite pulse width. However, simulation also showed that the time constants of the compensation should be reduced from the values that are necessary in continuous loops, a result which could mean that smaller capacitors are in order. A comparison of the hardware involved in the continuous and multiplexed configurations will indicate whether the objectives of reducing the number of components or achieving a more compact package through multiplexing were successful.

The compensation described in Section I is presently repeated in each of six separate control loops. To gain more insight into the feasibility of multiplexing, a detailed comparison of hardware in both configurations will be made assuming this six loop system. The present form of the compensation is an operational amplifier containing two differential amplifiers, with the appropriate dynamics for the control loop installed in the feedback of the operational amplifier. This feedback, or parallel, arrangement is frequently used to employ smaller

capacitors than would otherwise be the case, since for amplifiers with extremely high input impedance the effective capacitance is approximately the actual capacitance multiplied by the factor  $1 + \text{amplifier gain}$ . The RC filter for dynamics consists of the parallel combination of a 50 microfarad capacitor and a 20 megohm resistor in series with a 3 megohm resistor. Because these elements are located in the feedback of the amplifier, the form of the compensation includes the Laplacian operator  $s$  in both numerator and denominator, and specifically, is  $\frac{1 + s/.007}{1 + s/.001}$ . See Figure 18.

The simulation indicated that when this capacitor enters a particular loop, it must have the same voltage it had when it was switched out of that circuit. Therefore, some circuitry will normally be necessary to restore the proper value of voltage to the capacitor. Such circuitry is certainly possible, however, the problem is complicated by the absence of a ground point in the feedback of the operational amplifier. The voltage at the output of the amplifier is 15 volts; the voltage at the input is 6 volts. Circuitry designed to avoid a ground point is usually quite involved, and certainly seemed so in this case. Preliminary work involving design of such a circuit indicated that the requirement of memory ruled out the employment of the dynamics in the feedback of the compensation amplifier if the multiplexed system was to be even remotely competitive.

The alternative arrangement is to put the filter in the forward path of the compensation amplifier rather than the feedback. To realize



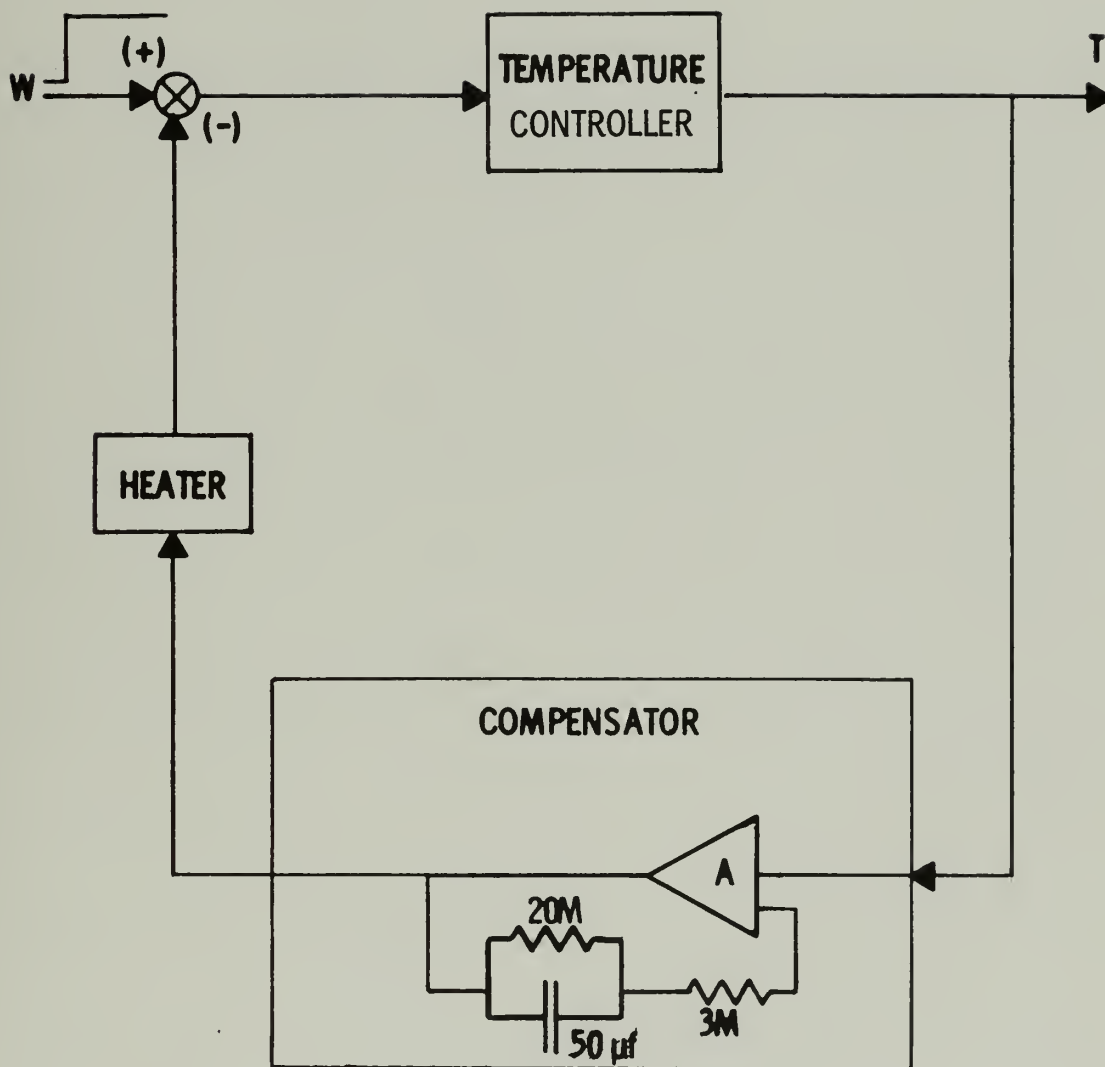


Fig. 18. Simplified block diagram of temperature control system showing present form of feedback compensation.

the desired transfer function in this configuration, the parallel combination of a resistor and a capacitor in series with another capacitor is required, with the series capacitor grounded. See Figure 19.

The heater consists of a two stage transistor amplifier with the load resistor connected to the collector of the second transistor. Transistor parameters are quite susceptible to changes in temperature and operating point. It would be desirable if such deviations from the design values had a negligible effect on the loop. The effect on the phase of a loop would be minimized if the resistive effect on the output terminals of the compensation amplifier were primarily that of the filter. For this situation to be true, the parallel combination of filter resistance and heater input resistance must be effectively just the resistance of the filter. In other words, the resistance of the filter must be less than about ten percent of the input resistance of the first transistor of the heater amplifier.

For the configuration in Figure 19, the filter transfer function is  $\frac{s + 1/RC_1}{s + 1/R(C_1+C_2)}$ . The desired response of the controller specifies the break frequencies  $1/RC_1$  and  $1/R(C_1+C_2)$ . The traditional disadvantage of forward path compensation now becomes evident. The RC product is fixed, and stability considerations suggest that the R be much less than the input resistance of the following transistor. Since conventional transistors have low input impedance, it appears as though the capacitors must be large, precisely the opposite of one of

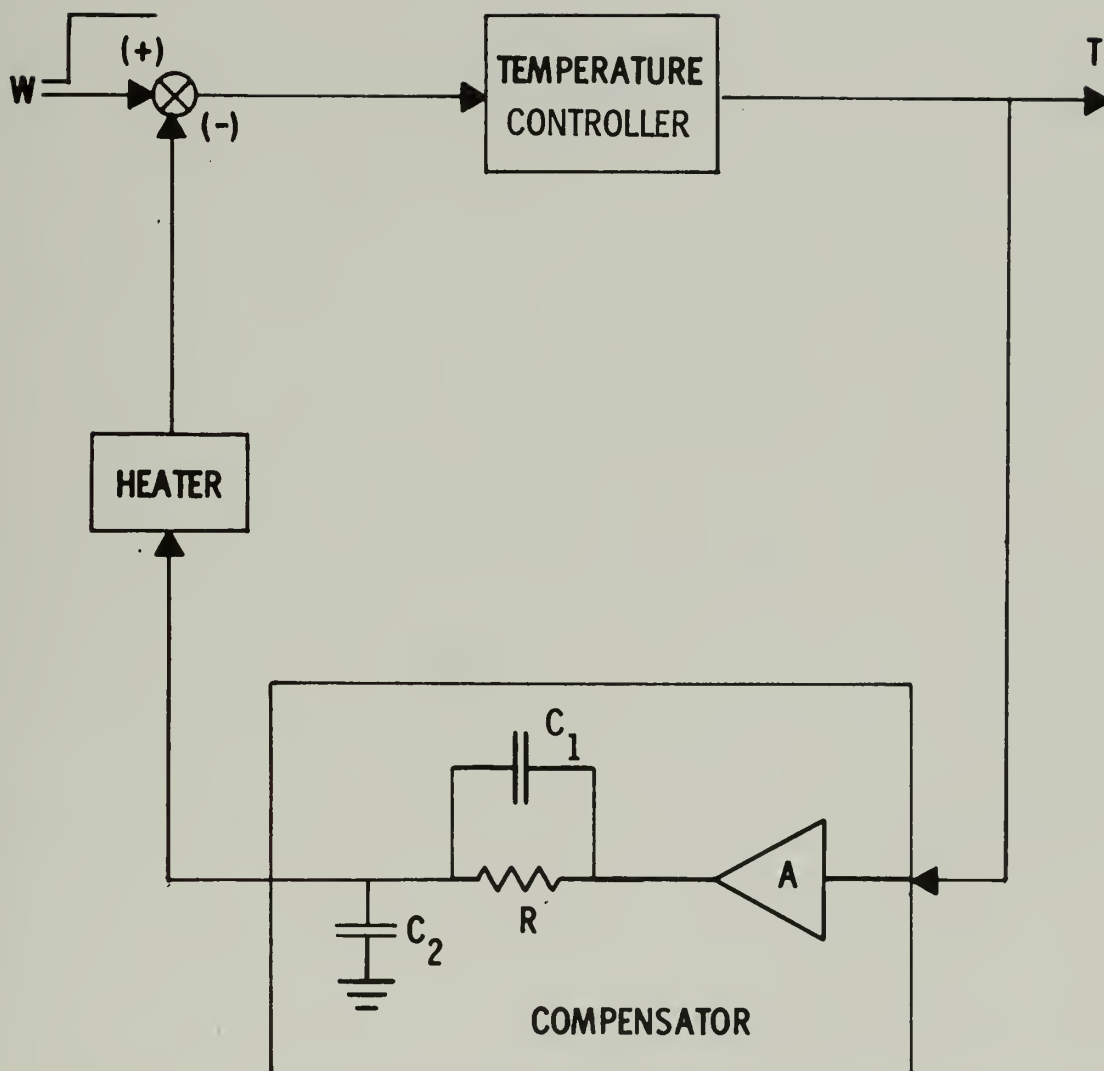


Fig. 19. Simplified block diagram of temperature control system showing proposed form of forward path compensation.

the intended objectives of this thesis. The luxury of the Miller effect, or factor of  $1 + \text{amplifier gain}$ , in reducing capacitor size is quite evident.

The solution to this problem lies in taking advantage of the modern field effect transistor. Historically one of the first types of transistors to be developed, only lately has it started to come into engineering usage. Field effect transistors have characteristics similar to vacuum pentodes. For this problem the important characteristic is the extremely high input impedance, a result of the normal biasing wherein the diode at the input is reverse-biased. A particular field effect transistor that has the capability of dissipating the power in the first stage of the heater amplifier is the Amelco U1714. The input diode leakage current for this transistor is 5 picoamps with 10 volts reverse-biasing. Visualize the current-voltage characteristics curve for a diode, passing through the origin and the point  $(-10 \text{ v}, -5 \text{ pa})$ . The exact input resistance of the FET will depend upon the biasing, but to give a reasonable estimate, one might define an average input resistance for this transistor of  $2 \times 10^{12}$  ohms. Then with this FET in the first stage of the heater amplifier, a very large resistor could be used in the filter.

The size of the filter resistor is limited by the leakage resistance of the parallel capacitor. The leakage resistance of a capacitor changes with variations in voltage and environmental conditions. Again for stability, the leakage resistance of the

capacitor should be an order of magnitude larger than the filter resistor. Polystyrene capacitors have very low leakage currents, however, a 1 microfarad polystyrene capacitor has dimensions of approximately 1 1/2" x 2". Since capacitor values for the filter are in the vicinity of 1 to 10 microfarads, polystyrene capacitors are unsuitable if one is interested in microminature packaging. Mylar capacitors also feature low leakage currents, but even they are about two-thirds the size of an equivalent polystyrene. The best compromise between small dimensions and low leakage currents at low frequencies appears to be the tantalum capacitor, which for the 1 to 10 microfarad range has leakage resistances of the order of 200 megohms.

To give the desired break frequencies, a suitable forward path RC filter would be the parallel combination of a 20 megohm resistor and a 7 microfarad capacitor in series with a 43 microfarad capacitor. Since the same size resistor would be used in both forward and feedback compensations, the total capacitance required remains the same. In essence, the field effect transistor enabled the forward path RC filter compensation to become competitive with feedback compensation.

Having equalized the initial conditions, so to speak, an investigation of the feasibility of time multiplexing will now begin. Two alternatives are available. The first is to attempt to time share both amplifier and RC filter in all loops, the original aim of the thesis. In the event that the requirement of restoring capacitor

voltages results in quite complicated circuitry, the second alternative is to multiplex just the amplifier. Recall that the reduction in time constants will permit using less capacitance in the latter configuration than is presently used in the continuous system.

Multiplexing this filter in each loop presents obvious disadvantages, since voltages must be restored on each capacitor. However, consideration might be given to sharing just the large series capacitor and the compensation amplifier. Then for each of the six loops two switches would be necessary for the amplifier and two more for the multiplexed capacitor. A switch must be installed adjacent to the parallel capacitor in order that it not discharge through the filter resistor during the hold period. A small capacitor must connect the input of the first stage field effect transistor to ground in order that constant heater voltage be maintained during the hold period. When the series capacitor is switched in, it would have to assume the voltage of the smaller hold capacitor. To equalize the voltage between the two capacitors, an amplifier must be available to provide the additional charge. After the voltage is equalized, it would be necessary to bypass this charging amplifier. Lastly, switching would be required to discharge the multiplexed capacitor before entry into the next loop. It may be argued that complete discharge is not necessary, or might not even be desirable, since the requirement is for the voltage of the multiplexed capacitor to be equalized with that of the hold capacitor. Thus some voltage comparison circuitry might



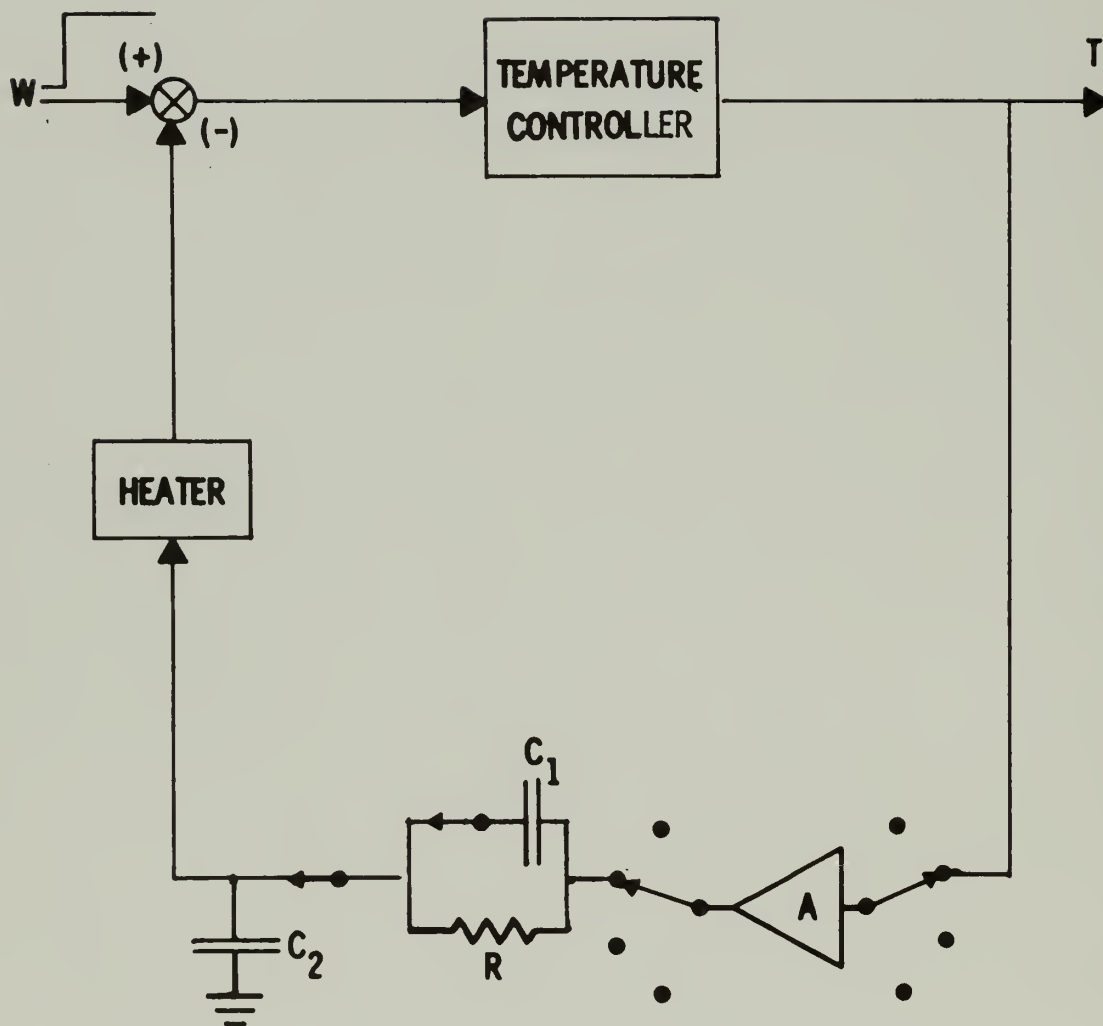


Fig. 20. Simplified block diagram of temperature control system illustrating multiplexing of compensation amplifier.



of identical controllers. The objective of improving reliability through reduction of components was not achieved to satisfaction. Although fewer amplifiers are needed, the amount of switching and circuitry exceeds that which had been desired.



## Section VI

### Summary and Evaluation

The purpose of this thesis was to study the feasibility of using a single compensation device with both gain and dynamics to stabilize several identical control loops. The ultimate objective was to improve reliability of the system through a reduction in the number of components and to reduce the space required for the system by eliminating redundant components. It was assumed that the form of the compensation was to be analog rather than digital. Not only does this assumption limit the form of the compensation which can be realized by amplifiers, resistors and capacitors; but also requires that the research be carried out in a manner that will permit practical implementation.

The usual sampled data analysis assumes that a signal is sampled during an infinitesimally small period of time, and that this value is held during the remainder of the sampling period. Impulse sampling of a multiloop system demands a considerable number of holds. The number may not be so important to a designer with digital logic available, but certainly is to the designer who is interested in a compact module with the minimum number of transistors, resistors and capacitors. In an attempt to reduce the amount of memory circuitry, a sampled data system with a finite pulse width was analyzed.

The compensation would consist of an amplifier for gain and an RC filter for phase. For a system with  $n$  loops, the compensation

would be available for a particular loop for a pulse duration of  $T$  seconds, then removed for  $(n-1)T$  seconds. The reasoning behind this approach was if the compensator were in each loop for a suitable length of time and if the break frequencies of the compensation were modified from the values used in the unshared system, then perhaps enough regulation could result from the time that the compensator spent in each loop. During switching the capacitors would be discharged to decouple the individual loops.

Analog computer simulation was used to determine the appropriate values of gain, break frequencies and pulse duration. The particular system modeled was a temperature regulator. A unique and extremely convenient procedure for magnitude scaling evolved during the modeling.

The computer simulations indicate that it is possible to multiplex compensation in several control loops, provided that when the compensation enters a loop it has very close to the same information that it did when it was switched out of that circuit. Even if a finite pulse width is used, the output of each controller must be reconstructed. For the loops to be decoupled a separate memory channel for each loop must be available in the compensator.

To complete this investigation, it was necessary to determine what hardware would be needed to implement a system compatible with the results of the simulation. It was shown that field effect transistors could be effectively employed to reduce the size of capacitors in RC filters when circumstances require that the filter be installed in the

forward path of the compensation amplifier. This concept could be useful in both continuous and multiplexed systems. Detailed comparison of component requirements for systems using unshared compensation, multiplexed compensation involving both gain and dynamics, and multiplexed compensation of gain only indicates that it is not feasible to time share analog compensation consisting of both amplifier and filter due to the requirement of memory for the dynamics of the compensation.

There might be occasions when it would be advantageous to multiplex the compensation amplifier. Time sharing just the amplifier still requires that the break frequencies of the compensation filter be adjusted from the values appropriate for a system without multiplexing. This modification consists of reducing the time constants of the filter, which implies that the RC product is decreased. Therefore, multiplexing the amplifier permits smaller capacitors to be used in the filter. For a system of six loops, the capacitance will be 20-25% of that in the unshared configuration. Advantages of such a design would be a significant decrease in the number of amplifiers and size of the capacitors. The disadvantage is that substantial switching is required, not only to switch the amplifier into each circuit but also to insure that all capacitors in each filter maintain their voltages during the hold period. The decision whether or not to use multiplexing should depend upon the particular circumstances examined in the light of the above information.



## Section VII

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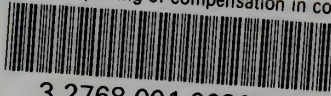






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